(1) The model makes use of the local Boltzmann theory even in the nonlocal case.

(2) The theory is valid only for constant relaxation times neglecting any quantization effects.[7]

(3) The electrons suffer not only diffuse scattering (a rough estimation suggests a 20 to 70% fraction of specular scattering).

(4) The measuring is disturbed by the fact that the surface is not absolutely smooth and, therefore the field intensity is not constant [L_D (300°K) = 0.03 μ].

Although our measurements are not in quantitative agreement with the theory, our experimental work confirms very well the predictions made for the nonlocal case by the surface-transport theory.

I wish to express my sincere appreciation to Professor H. Welker and Dr. H. Weiss, who made many valuable suggestions. I am indebted to Dr. H. Pfleiderer for several discussions.

[7] R. F. Greene, Surface Sci. 2, 101 (1964).

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Oscillatory Photoconductivity of GaSb under a Magnetic Field*

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The spectral oscillation of intrinsic photoconductivity in GaSb has been studied under applied magnetic fields. There is a shift of the minima to higher photon energies corresponding to an increase of the energy gap under a magnetic field. More strikingly, additional structures appear in the spectrum. The oscillation is more pronounced with longitudinal magnetic fields than with transverse fields. The spectrum is interpreted by considering transitions between Landau bands of the conduction band and those of the valence band. Under a longitudinal field, a minimum in photoconductivity may occur when the energy of photoexcited electrons, after the emsision of LO phonons if the initial energy is sufficiently high, is close to a Landau level. As expected, the oscillation is weaker under a transverse field and may even appear to be reversed.

I. INTRODUCTION

Spectral oscillations of photoconductivity with a frequency interval related to the LO phonon energy have been studied extensively in recent years.[1] The condition for the oscillations to occur is that the time τ_{op} required for the emission of optical phonons is short in comparison with the lifetime τ_l of the carriers. Under this condition, the carriers will have for the most part of their lifetime an energy limited to the range below $\hbar\omega_0$, the energy of a longitudinal optical phonon. Another necessary condition is that within this range the energy of carriers does not change too fast in comparison with the carrier lifetime. Otherwise, the photogenerated carriers would be all thermalized. Under the condition

$$\tau_\epsilon > \tau_l > \tau_{op},$$

τ_ϵ being an energy relaxation time, the energy distribution of photogenerated carriers depends on $h\nu$, oscillating within the range below $\hbar\omega_0$[2]. This effect is basically the origin of the oscillatory photoconductivity.

An oscillation can be produced through the energy dependence of mobility as well as through an energy dependence of carrier lifetime. The oscillation of intrinsic photoconductivity in GaSb was attributed to the mobility variation in zero magnetic field.[2] If a narrow energy distribution of carriers is indeed obtained under photoexcitation, additional structure may appear in the spectrum under an applied magnetic field as the carrier energy passes through the Landau levels. We are not concerned with the effect of magneto-oscillations in absorption which is observable only for sufficiently thin samples and is easily avoided in measurements of intrinsic photoconductivity. In fact, most of the additional structures do not coincide with the expected variation of absorption. A magnetic field produces a variation of density of states which may cause the scattering to vary with the carrier energy, introducing consequently structures in the photoconductive spectrum. Impurity states and exciton states associated with a higher Landau level are unstable and should be much less effective for recombination than those associated with the lowest Landau levels. It is therefore unlikely that the effect is connected with the carrier lifetime.

We have measured the intrinsic photoconductivity of p-type GaSb under magnetic fields up to 80 kG, in

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[1] H. Y. Fan, in *Proceedings of the Ninth International Conference on the Physics of Semiconductors, Moscow, 1968* (Publishing House Nauka, Leningrad, 1968), p. 135. References are given to previous work.

[2] M. A. Habegger and H. Y. Fan, Phys. Rev. Letters 12, 99 (1964).

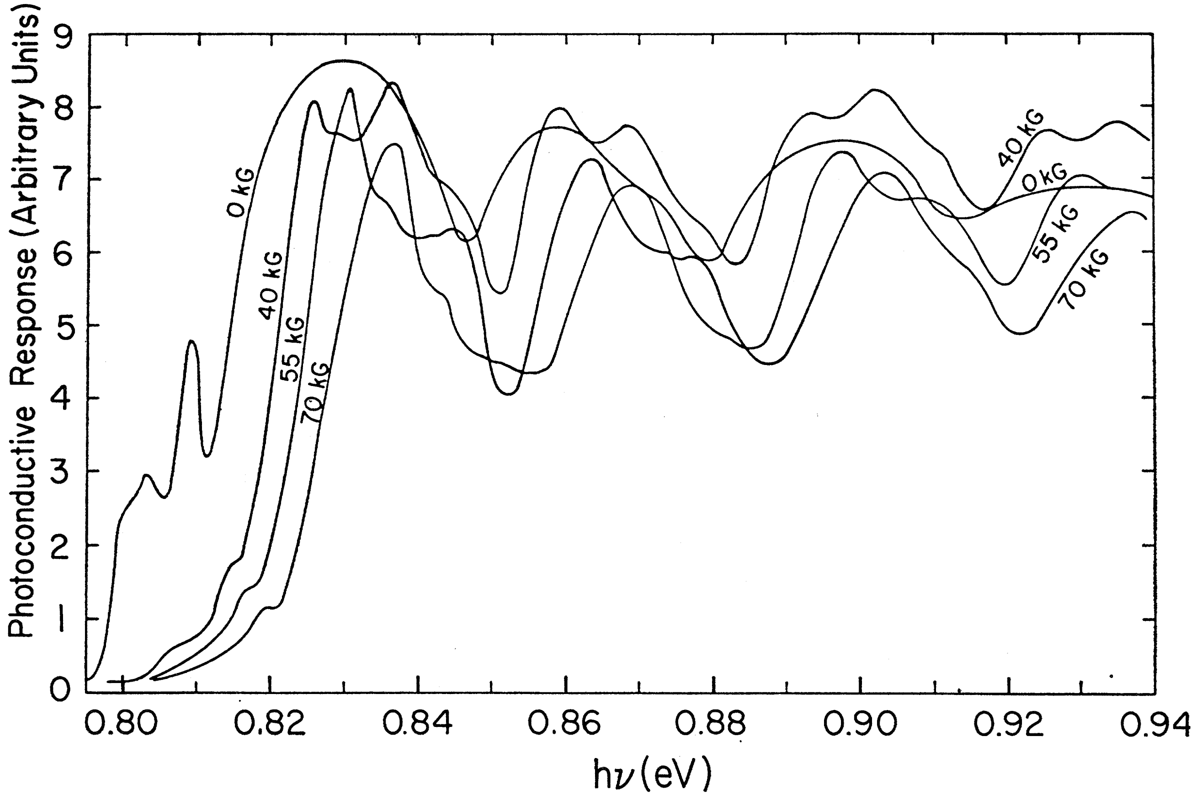

FIG. 1. Spectra of photoconductivity in longitudinal magnetic field, **H**∥[110], polarization **E**⊥**H**. A shoulder at ~0.869 eV for $H=0$ and a small dip at ~0.875 eV for $H=55$ kG are due to transitions from the light hole band. Other minima are given by transitions from the heavy hole band.

longitudinal and transverse configurations. The spectrum shows the Landau level effect as well as the effect of optical phonons. The results are interpreted in terms of the band structure and the transport theory in magnetic field. Some preliminary measurements had been made by LeComber in this laboratory, the results of which were reported previously.[1]

II. EXPERIMENTAL DETAILS

The samples studied were not intentionally doped. The hole concentration in the exhaustion range was about 2×10^{17} cm^{-3}. The photoconductivity spectra were measured with a Perkin-Elmer model-E_1 monochromator. Sylvania Sun Gun was used as the light source and the light was chopped at ~91 Hz. The light was either unpolarized or polarized by a HR Polaroid. Polarization was either parallel or perpendicular to the magnetic field.

The magnetic field up to 80 kG was provided by a superconducting solenoid. Measurements were made at about 1.7°K, in order to eliminate the noise due to the bubbling of liquid helium above the λ point.

In transverse measurements, the sample was placed with its broad surface either perpendicular to the magnetic field (horizontal position) or parallel to the magnetic field (vertical position). In the horizontal position, light was parallel to the magnetic field. In the vertical position, light was perpendicular to the magnetic field. In either position the dc voltage was applied to the length of the sample, which is parpendicular to the magnetic field. There might be some photoelectromagnetic effect in the latter case. Measurements with samples in the vertical position were made only for comparison. Detailed studies were made with samples in the horizontal position.

III. EXPERIMENTAL RESULTS

In the absence of an applied magnetic field, the photoconductive spectrum shows an oscillation with minima separated by 0.0336 eV. Under an applied magnetic field, either longitudinal or transverse, the minima shift to higher photon energies along with the long-wavelength edge of the spectrum, as shown by Fig. 1 for several values of a longitudinal field. These minima will be called the main minima. The energy shift corresponds to the increase of energy gap under a magnetic field.

A. Longitudinal Magnetic Field

As can be seen in Fig. 1, additional minima appeared in the spectrum under an applied magnetic field. The energies of minima observed at various fields are plotted in Figs. 2 and 3 for light polarized with $\mathbf{E}\|\mathbf{B}$ and $\mathbf{E}\perp\mathbf{B}$, respectively. For the different polarizations, minima were observed at somewhat different energies which may be expected from a difference in selection rules for transitions. Measurements were made also with different crystal orientations. There was little change in the spectrum showing that the anisotropy of the energy bands is too small to be resolved by the measurements.

Figure 2 and 3 show that there are several series of additional minima, each with the same energy spacing as that of the set of main minima. There are also some minima below the one-phonon main minimum which are not repeated at higher energies. It is shown later that the interpretation is the following: (1) A main minimum appears when the electron energy after the emissipn of optical phonons coincides with the $n=0$ Landau level. (2) A minimum in an additional series occurs when the final electron energy coincides with the energy of a higher Landau level, higher than the $n=0$ level by less than $\hbar\omega_0$. These minima are given by transitions from the heavy hole band. (3) The extra minima which do not form series are due to transitions from the light hole band.

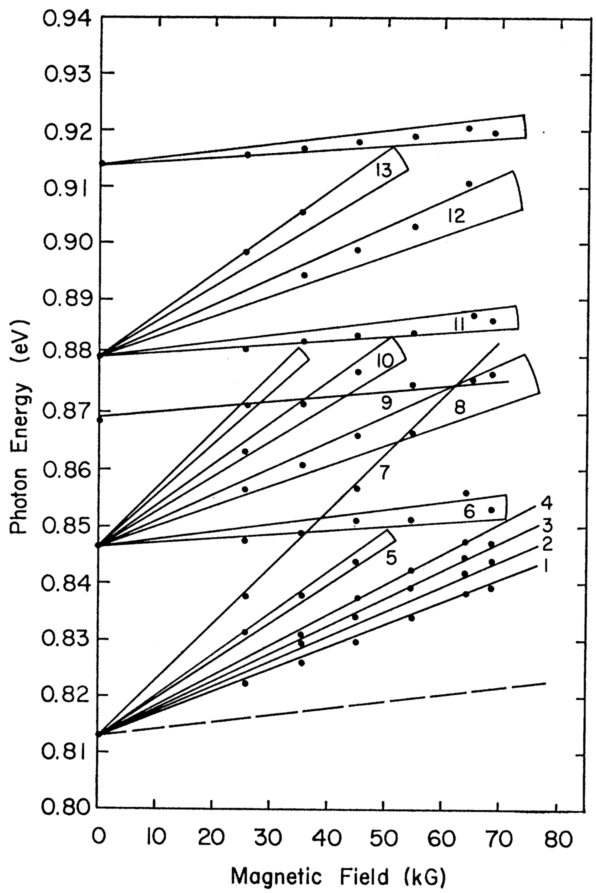

FIG. 2. Minima in photoconductivity versus a longitudinal magnetic field, $\mathbf{H}\|[112]$, $\mathbf{E}\|\mathbf{H}$. Sample $27A21$. The lines are calculated. Each set of two lines connected at the end give limits of several closely spaced minima.

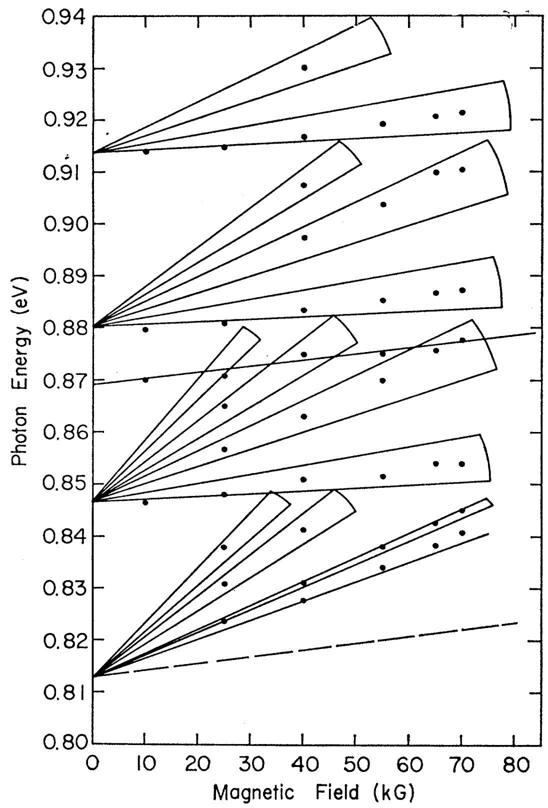

FIG. 3. Minima in photoconductivity versus a longitudinal magnetic field, $H\|[110]$, $E\perp H$. Sample $70B\gamma$.

The structures become more pronounced with increasing magnetic field. The effect on the main minima is clearly seen in Fig. 1. The depth of an additional minimum produced by a higher Landau level is plotted versus $1/\hbar\omega_c$ in Fig. 4, where ω_c is the cyclotron frequency. The depth of a minimum refers to the drop of signal in terms of the value obtained by joining the two neighboring maxima. The plot resembles the relation $\exp(-2\pi\Gamma/\hbar\omega_c)$ expected for the de Haas–Schubnikov oscillation, where Γ is the broadening of energy levels.[3,4] The plot gives $\Gamma\sim3.5\times10^{-3}$ eV. With $\Gamma=\hbar/2\tau$ where τ is a relaxation time, we get $\omega_c\tau\sim2.5$. The value of τ in this rough estimate is associated with level broadening. It is reassuring that the value has the order of magnitude of the relaxation time characterizing the mobility.

B. Transverse Magnetic Field

The spectrum shows also additional structure under a transverse magnetic field. Comparison of the two spectra

[3] R. Kubo, S. Miyake, and N. Hashitsume, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1965), Vol. 17, p. 269.

[4] A. D. Brailsford, Phys. Rev. **149**, 456 (1966).

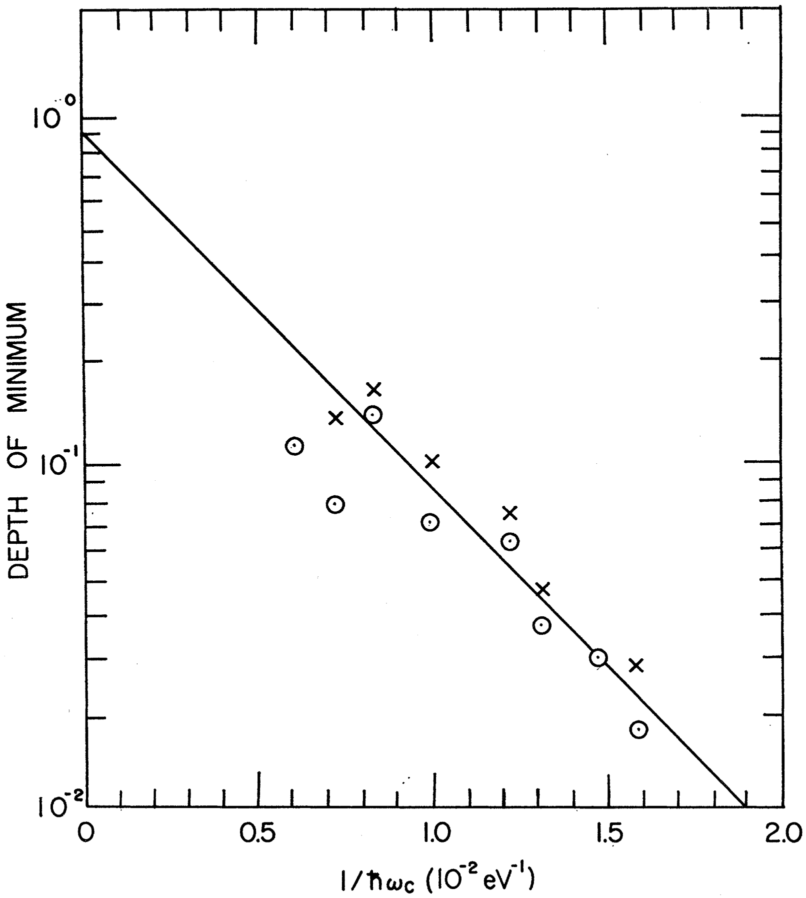

FIG. 4. Relative amplitude of additional minima versus a longitudinal magnetic field. Points × include first two minima without phonone mission. Points ○ represent the minima with the emission of one LO phonon. Sample 27A24.

in Fig. 5 indicates that the spectrum under transverse field appears to show additional maxima corresponding to the additional minima in the spectrum for longitudinal field. Also, both the main and additional minima under a transverse field are much less pronounced than in the longitudinal field. Figure 6 shows the depths of two main minima for various transverse fields. They stay nearly constant with increasing field. Even a decrease was observed in some measurements.

With the sample in a vertical position, the spectrum was somewhat different for different light polarizations. The situation is similar to the case with longitudinal field. Polarization made no difference with the sample in horizontal position since the polarization was always perpendicular to the magnetic field. No plot of the type of Fig. 2 or Fig. 3 is shown for the transverse field. The positions of the minima are not clearly defined as in the case of longitudinal field.

IV. INTERPRETATION

Let z be the direction of magnetic field. The measured resistance of the sample gives

$$\rho_{zz}=1/\sigma_{zz} \tag{1}$$

for a longitudinal field, and gives

$$\rho_{xx}=\sigma_{xx}/(\sigma_{xx}^2+\sigma_{xy}^2) \tag{2}$$

for a transverse field. ρ is the resistivity and σ is the conductivity. The photoconductive signal, the change of resistance produced by light, gives

$$\Delta\rho_{zz}=-\Delta\sigma_{zz}/\sigma_{zz}^2 \tag{3}$$

in the case of longitudinal fields. Photoconductivity in a transverse field gives

$$\Delta\rho_{xx}=\frac{(\sigma_{xy}^2-\sigma_{xx}^2)\Delta\sigma_{xx}-2\sigma_{xx}\sigma_{xy}\Delta\sigma_{xy}}{(\sigma_{xx}^2+\sigma_{xy}^2)^2}. \tag{4}$$

In our measurement, the dark conductivity was dominated by impurity conduction for which $\sigma_{xx}\gg\sigma_{xy}$. Therefore,

$$\Delta\rho_{xx}\simeq-\Delta\sigma_{xx}/\sigma_{xx}^2. \tag{5}$$

In de Haas–Schubnikov experiments, usually $\sigma_{xy}\gg\sigma_{xx}$ and σ_{xy} does not oscillate strongly with magnetic field. Therefore, the oscillation of ρ_{xx} measured in transverse fields gives the oscillation of σ_{xx}, whereas the oscillation of ρ_{zz} measured in longitudinal fields gives the oscillation of $1/\sigma_{zz}$. In photoconductivity, measurement in

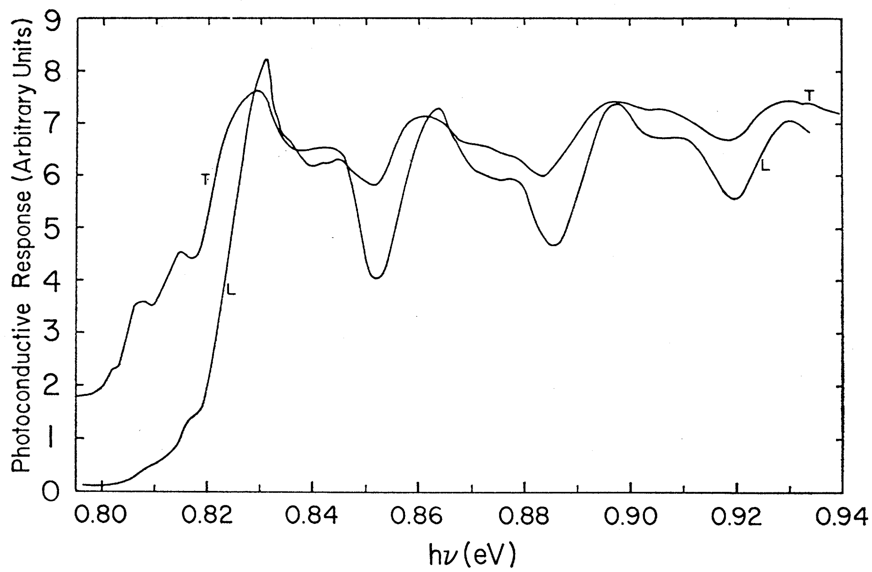

FIG. 5. Longitudinal (L) and transverse (T) spectra measured on the same sample $70B\gamma$. In both cases, $H=55$ kG, $\mathbf{H}\|[110]$, $\mathbf{E}\|\mathbf{H}$. The transverse measurement was made with the sample in horizontal position.

transverse fields shows the variation of $\Delta\sigma_{xx}$ which appears to be analogous to the de Haas–Schubnikov measurement, even though for a different reason. With longitudinal fields, however, photoconductivity shows the variation of $\Delta\sigma_{zz}$ instead of that of $1/\sigma_{zz}$.

A. Longitudinal Magnetic Field

With a nearly monoenergetic distribution of photoexcited electrons, carrier scattering is high when the energy is close to a Landau level where the density of states is large. Therefore, we expect $\Delta\sigma_{zz}$ and photoconductivity to show a minimum when the final energy of photoexcited electrons is close to a Landau level. The conduction band of GaSb has an effective mass $m_e=0.047\ m$ [5] and an effective g value $g_c=6.5$.[6] The electron energy in the conduction band is given by

$$\epsilon\begin{pmatrix}a\\ \ ,n\\ b\end{pmatrix}=\frac{m}{m_e}(n+\tfrac{1}{2})\pm\tfrac{1}{4}g_c\,, \qquad (6)$$

where the energy is expressed in units of $\hbar eH/mc$. The parameters of the valence band are[7]: $A=-11$, $B=-6$, $|C|-11$. From these values we obtain

$$\gamma_1=-A=11\,,\quad \gamma_2=-\tfrac{1}{4}B=3\,,$$
$$\gamma_3^2=\tfrac{1}{2}(\tfrac{1}{3}C^2+B^2)=4.4. \qquad (7)$$

Two quantities related to these parameters are

$$\gamma'=\gamma_3(\gamma_2-\gamma_3)[(3\cos^2\theta-1)/2]^2\,, \qquad (8)$$

$$\gamma''=\tfrac{2}{3}\gamma_3+\tfrac{1}{3}\gamma_2+\tfrac{1}{6}(\gamma_2-\gamma_3)[(3\cos^2\theta-1)/2]^{1/2}\,, \qquad (9)$$

where θ is the angle between the magnetic field and the z axis in the $(1\bar{1}0)$ plane. Another quantity, κ, may be obtained from the approximate formula

$$\kappa=\gamma_3+\tfrac{2}{3}\gamma_2-\tfrac{1}{3}\gamma_1-\tfrac{2}{3}\,. \qquad (10)$$

The hole energy for $k_z=0$ may be calculated using the following approximate solutions[8]:

$$\epsilon(a^{\pm},n)=\gamma_1 n-(\tfrac{1}{2}\gamma_1+\gamma'-\tfrac{1}{2}\kappa)$$
$$\pm\{[-\gamma'n+(\gamma_1+\tfrac{1}{2}\gamma'-\kappa)]^2+3\gamma''^2n(n-1)\}^{1/2}\,, \qquad (11a)$$

$$\epsilon(b^{\pm},n)=\gamma_1 n-(\tfrac{1}{2}\gamma_1-\gamma'+\tfrac{1}{2}\kappa)$$
$$\pm\{[\gamma'n+(\gamma_1-\tfrac{1}{2}\gamma'-\kappa)]^2+3\gamma''^2n(n-1)\}^{1/2}\,, \qquad (11b)$$

where $+$ and $-$ signs refer to light and heavy holes, respectively. n is an integer which begins with 0 for the $+$ sign and begins with 2 for the $-$ sign.

We assume that k_z contributes $\hbar^2k_z^2/2m^*$ to the energy where m^* is an appropriate effective mass for the energy band. Thus the photon energy for transitions is given by

$$h\nu=E_g+\epsilon_e(S',n+\Delta n)+\epsilon_h(S^{\pm},n)$$
$$+\tfrac{1}{2}(1/m_{\pm}+1/m_e)\hbar^2k_z^2\,, \qquad (12)$$

where $E_g=0.813$ eV is the energy gap and S stands for a or b. ϵ_e and ϵ_h are, respectively, the electron energy and the hole energy at $k_z=0$ which are given by (6) and (11) in units of $\hbar eH/mc$. At $k_z=0$, transitions are allowed for $S=a$, $S'=b$, $\Delta n=0$ and for $S=b$, $S'=a$, $\Delta n=-2$ when the light is polarized with electric vector parallel to the magnetic field. These transitions will be denoted as $a^{\pm}(n)b^c(n)$ and $b^{\pm}(n)a^c(n-2)$, respectively. With light polarized perpendicular to the magnetic field, the conditions for allowed transitions at $k_z=0$ are $S'=S$ and $\Delta n=0$ or $\Delta n=-2$, i.e., the allowed transitions

[5] S. Zwerdling, B. Lax, K. J. Button, and L. M. Roth, J. Phys. Chem. Solids 9, 320 (1959).

[6] E. J. Johnson, I. Filinski, and H. Y. Fan, in *Proceedings of International Conference on the Physics of Semiconductors, Exeter, 1962* (The Institute of Physics and The Physical Society, London, 1962), p. 375.

[7] R. A. Stradling, Phys. Letters 20, 217 (1966).

[8] L. M. Roth, B. Lax, and S. Zwerdling, Phys. Rev. 114, 90 (1959).

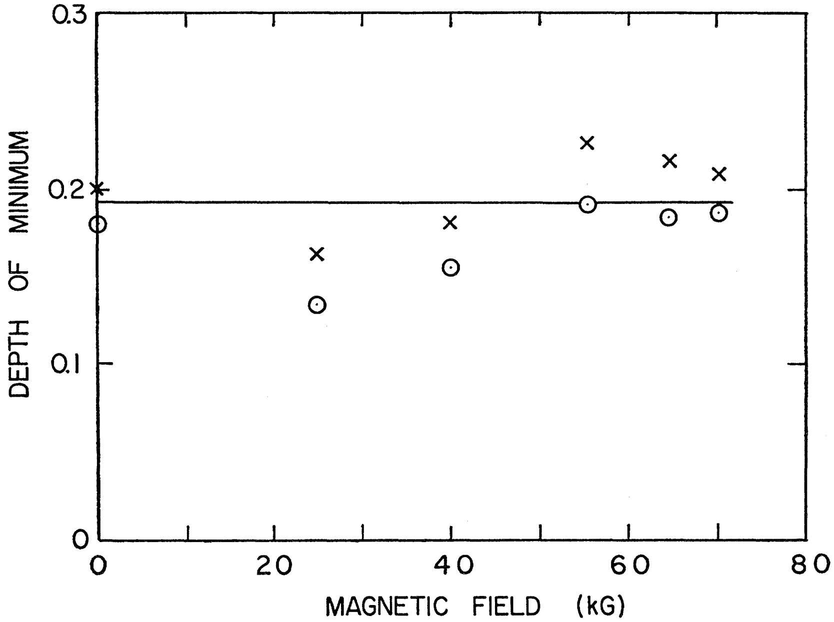

FIG. 6. Relative magnitude of main minima versus a transverse magnetic field. The points × and ○ represent the minima with the emission of one and two phonons, respectively. Sample $70B\gamma$ in horizontal position.

are $a^{\pm}(n)a^c(n)$, $a^{\pm}(n)a^c(n-2)$, $b^{\pm}(n)b^c(n)$, and $b^{\pm}(n)b^c(n-2)$. For simplicity, we overlook the restriction $k_z=0$ for the selection rules.

After quickly emitting LO phonons, the photoexcited electrons spend most of their lifetime with an energy ϵ_f in the range of $0\leq\epsilon_f<\hbar\omega_0$ above the lowest Landau level:

$$\begin{aligned}\epsilon_f&=\epsilon_e(S',n+\Delta n)+\hbar^2k_z^2/2m_e-l\hbar\omega_0-\epsilon_e(a,0)\\&=\epsilon_e(S',n+\Delta n)-\epsilon_e(a,0)-l\hbar\omega_0\\&\quad+[h\nu-E_g-\epsilon_e(S',n+\Delta n)-\epsilon_h(S,n)]\\&\qquad/(1+m_e/m_\pm)\,,\end{aligned}\tag{13}$$

where l is an integer. The energy dispersion of optical phonons is neglected, since the dispersion is small and the effective range of phonon wave vectors[9] is also small. The relation (13) is plotted in Fig. 7 for some of the transitions for $H=45$ kG, $\mathbf{H}\|[112]$ or $[110]$, and $\mathbf{E}\|\mathbf{H}$. For a given transition, ϵ_f increases linearly with $h\nu$ dropping abruptly upon reaching $\hbar\omega_0$. The process is repeated. The lines corresponding to various transitions from a given hole band are parallel to each other. Lines for transitions from the heavy hole band have a slope of $(1+m_e/m_-)^{-1}$ and a period of $(1+m_e/m_-)\hbar\omega_0$ $=0.0336$ eV. These lines are close to each other since

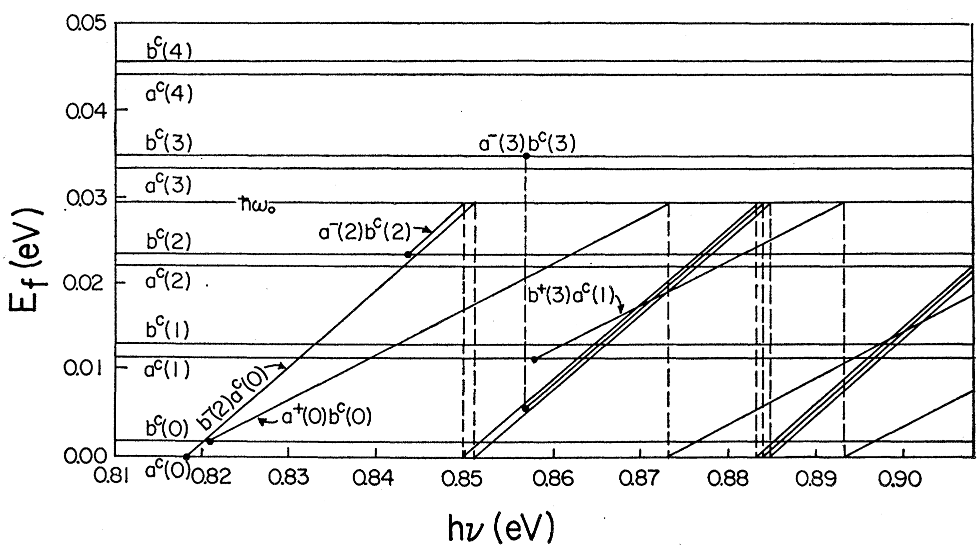

FIG. 7. Relation between ϵ_f and $h\nu$ calculated for $H=45$ kG. For clarity, only some of the allowed transitions are shown.

[9] R. C. Enck, A. S. Saleh, and H. Y. Fan, Phys. Rev. **182**, 790 (1969).

the separations between Landau levels of the heavy hole band are small. The lines for transitions from the light hole band have a slope of $(1+m_e/m_+)^{-1}$ and a period of 0.056 eV. These lines are not so close to each other.

Such a plot was made including all possible transitions. Horizontal lines were drawn to correspond to the various Landau levels. Intersections of the lines with the plots of ϵ_f gave the photon energies at which minima might occur in photoconductivity. The observed minima were then identified. The results are given in Table I. The first column gives the numbering of the observed minima. The third column gives the Landau level which coincides with ϵ_f. The Landau bands of conduction and valence bands between which the transitions occur are given in columns 4 and 5. For several of the observed minima, more than one possibilities are listed in column 3 and also in columns 4 and 5, indicating that in the neighborhood of the observed minima several minima may be expected which could be difficult to resolve. In Figs. 2 and 3, each set of points is mostly enclosed by two lines which show the spread of the estimated possible minima.

The results in Table I bring out the following points: (1) A minimum may be expected from $k_z\sim 0$ transitions to a Landau level which is within $\hbar\omega_0$ of the edge of the conduction band. The minima designated by 2, 3, 4, 7 are of this type. Such minima may have corresponding maxima in magnetoabsorption. (2) Minima can be produced by $k_z\neq 0$ transitions when transitions are made to states which coincide in energy with a Landau level in the range of $\hbar\omega_0$ from the band edge. The minima 1, 5, 7 might have some contribution from this mechanism. In principle, this kind of minima may also be given by transitions from the light hole band but such minima were not found in our results. (3) Transitions for $k_z\neq 0$ can give minima with the collaboration of phonon emission, the condition being that ϵ_f coincides with a Landau level. Minima due to the lowest Landau level, $a^c(0)$ or $b^c(0)$, and $k_z\neq 0$ transitions from the heavy hole band are called the main minima (6 and 11 in Table I). They form a series with the emission of various numbers of phonons. The minima, 8, 10, 12, 13, are produced by the higher Landau levels, $a^c(1)$, $b^c(1)$, $a^c(2)$, $b^c(2)$. Minima due to different heavy hole Landau bands are close in photon energy and were not resolved in the measurement. (4) The minimum 9 is associated with transitions from the light hole band. It was observed at $H=0$, 10, 55 kG. It was very weak. Only the minimum with emission of one phonon was detected. (5) The minimum 7 is also due to the light hole band with $k_z\neq 0$ and/or $k_z=0$ transitions. Figure 2 shows that this minimum was observed even at $h\nu$ larger than that of the one-phonon main minimum This is understandable since the Landau level, $a^c(1)$, concerned lies within $\hbar\omega_0$ from the band edge for the magnetic fields used.

TABLE I. Identification of minima observed under a longitudinal magnetic field $H=45$ kG, $\mathbf{H}\|[110]$ or $[112]$, with polarized light $\mathbf{E}\|\mathbf{H}$.

	Number of phonons emitted	LL which coincides with ϵ_f	Transition: Conduction band	Transition: Valence band		k_z
1	0	$a^c(1)$	$a^c(0)$	$b^-(2)$		$\neq 0$
2	0	$a^c(1)$	$a^c(1)$	$b^-(3)$		0
3	0	$b^c(1)$	$b^c(1)$	$a^+(1)$		0
4	0	$a^c(0)$	$a^c(0)$	$b^+(2)$		0
5	0	$a^c(2)$ or $b^c(2)$	$a^c(n)$	$b^-(n+2)$	$n=0, 1$	$\neq 0$
		$a^c(2)$ or $b^c(2)$	$a^c(2)$	$b^-(4)$		0
		$b^c(2)$	$b^c(2)$	$a^+(2)$		0
6	1	$a^c(0)$ or $b^c(0)$	$a^c(n)$	$b^-(n+2)$	$n=0$ to 2	$\neq 0$
			$b^c(2)$	$a^+(2)$		$\neq 0$
7	0	$a^c(1)$	$a^c(1)$	$b^+(3)$		0
			$a^c(0)$	$b^+(2)$		$\neq 0$
8	1	$a^c(1)$ or $b^c(1)$	$a^c(n)$	$b^-(n+2)$	$n=0$ to 3	$\neq 0$
			$b^c(n)$	$a^-(n)$	$n=2, 3$	$\neq 0$
9	1	$a^c(0)$	$b^c(0)$	$a^+(0)$		$\neq 0$
10	1	$a^c(2)$ or $b^c(2)$	$a^c(n)$	$b^-(n+2)$	$n=0$ to 4	$\neq 0$
			$b^c(n)$	$a^-(n)$	$n=2$ to 4	$\neq 0$
11	2	$a^c(0)$ or $b^c(0)$	$a^c(n)$	$b^-(n+2)$	$n=0$ to 5	$\neq 0$
			$b^c(n)$	$a^-(n)$	$n=2$ to 5	$\neq 0$
12	2	$a^c(1)$ or $b^c(1)$	$a^c(n)$	$b^-(n+2)$	$n=0$ to 6	$\neq 0$
			$b^c(n)$	$a^-(n)$	$n=2$ to 6	$\neq 0$
13	2	$a^c(2)$ or $b^c(1)$	$a^c(n)$	$b^-(n+2)$	$n=0$ to 7	$\neq 0$
			$b^c(n)$	$a^-(n)$	$n=2$ to 7	$\neq 0$

It is seen that the observed minima in photoconductivity can be correlated with the approximately calculated Landau levels. Pidgeon and Brown[10] have applied a more accurate method in the calculation for InSb. For GaSb with a much larger energy gap, the approximation we used seems to be adequate for the interpretion of the results. Recently, Adachi[11] calculated the band parameters of GaSb using the method of Pidgeon and Brown and the magneto-absorption data obtained by Zwerdling *et al.*[5] He obtained $\gamma_1=14.5$, $\gamma_2=\gamma_3=4.5$ (spherical symmetry of valence band was assumed) and $\kappa=2.7$ with $Eg=0.813$ eV, $m_e=0.045m$, $g=-6.5$, and $m_+=0.041m$. The values fit our experimental results less satisfactorily. In particular, we do not have a line which would correspond to the $a^+(2)a^c(0)$, $k_z=0$ transitions. Adachi assigned the observed absorption peak at 0.8303 eV to a $a^+(2)a^c(0)$ transitions whereas this peak should perhaps be assigned to $b^-(3)b^c(1)$, $a^+(1)a^c(1)$, or $a^-(3)a^c(1)$. We note that the value of γ_1 obtained by Adachi is significantly larger than the value $\gamma_1=11.0$ ± 0.6, obtained by Stradling[7] from cyclotron resonance of holes and the theoretical value $\gamma_1=11.15$ obtained by Higgenbotham *et al.*[12]

[10] C. R. Pidgeon and R. N. Brown, Phys. Rev. **146**, 575 (1966).
[11] E. Adachi, J. Phys. Chem. Solids **30**, 776 (1969).
[12] C. W. Higginbotham, F. H. Pollak, and M. Cardona, in *Proceedings of the Ninth International Conference on the Physics of Semiconductors, Moscow, 1968* (Publishing House Nauka, Leningrad, 1968), p. 57.

B. Transverse Magnetic Field

The conductivity tensors under an applied magnetic field are calculated by integrating over electronic states. The carrier distribution function, $f(\epsilon)$ is involved only in the form of $df/d\epsilon$ or $f(1-f)$ in the integrand.[3,13] For the case of a Fermi distribution at a low temperature, the distribution function introduces into the integrand approximately a δ function, $\delta(\epsilon-\zeta)$, where ζ is the Fermi energy under the applied field. In photoconductivity where the photoexcited carriers may be considered as monoenergetic and the excitation is not too strong, we have $f\propto\delta(\epsilon_f)/\rho_f$ and $(1-f)\sim 1$ where ρ_f is the density of states at the final energy ϵ_f. Thus we may expect that approximately

$$\Delta\sigma_{xx}\propto\sigma_{xx}{}^F(\epsilon_f)/\rho_f\,, \tag{14}$$

$$\Delta\sigma_{zz}\propto\sigma_{zz}{}^F(\epsilon_f)/\rho_f\,, \tag{15}$$

where $\sigma^F(\epsilon_f)$ is the conductivity of degenerate carriers with the Fermi energy equal to ϵ_f. The quantity ρ_f exhibits a maximum whenever ϵ coincides with a Landau level. At high magnetic fields, $\sigma_{zz}{}^F$ gives minima whereas $\sigma_{xx}{}^F$ gives maxima near Landau levels, therefore, ρ_f enhances the oscillation of photoconductivity in a longitudinal field and counteracts the effect of $\sigma_{xx}{}^F$ for the oscillation in a transverse field. This is consistent with the fact that oscillations observed in transverse fields were less pronounced than those observed in longitudinal fields and that sometimes the additional structure appeared even reversed as indicated by Fig. 5.

The above consideration applies to high magnetic fields with $\omega_c\tau\gg 1$. In a weak field with $\omega_c\tau\gtrsim 1$, the maxima of $\sigma_{xx}{}^F$ near Landau levels involve the lateral drift motion due to scattering among quantized orbits and are more uncertain than the minima in $\sigma_{zz}{}^F$ which are mainly the effect of the distribution of electronic states on scattering. This situation can be seen in the results of de Haas–Schubnikov effect measurements on n-type GaSb.[14] With $\omega_c\tau\gtrsim 2$, the ratio of observed oscillation amplitudes of transverse and longitudinal magnetoresistances was smaller than expected from the theory for $\omega_c\tau\gg 1$. It would be desirable to extend the photoconductivity measurements to higher magnetic fields but it should be borne in mind that only Landau levels within $\hbar\omega_0$ of the band edge produce structures in the spectrum.

[13] S. S. Shalyt and A. L. Efros, Fiz. Tverd. Tela 5, 1233 (1962) [English transl.: Soviet Phys.—Solid State 4, 903 (1962)].

[14] W. M. Becker and H. Y. Fan, in *Proceedings of the Seventh International Conference on the Physics of Semiconductors, Paris, 1964* (Dunod Cie., Paris, 1964), p. 663; T. O. Yep and W. M. Becker, Phys. Rev. 144, 741 (1966).

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Statistical Theory of Localized States of a Many-Impurity Crystal*

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A statistical theory of energy levels and position probability is derived for a one-demensional, statistically dilute, many-impurity crystal. The equations are evaluated for a simple model. It is shown that two impurity bands are generated, one above and one below the host band, each merging without a gap onto the host band. If U is greater than (less than) zero, the upper (lower) band is more dense. Each impurity-level electron is shown to be distributed in a wavelike manner along the impurities. The equations are derived for impurities differing both in their self-energy and in their nearest-neighbor coupling from the host monomers.

I. INTRODUCTION

IT is well known that impurities in crystals result in the appearance of so-called localized "impurity states," separated from the main band by a gap. The conditions for the appearance of such localized states and their location can be determined by the Green's-function method of Koster and Slater.[1–3] A very similar situation occurs for excited states in molecular crystals, where impurities again generate, under specified conditions, localized levels in the neighborhood of an exciton band.[4,5] Lifshitz[6] considers a disordered system, i.e., a nonregular impurity distribution, and calculates the spectral density by expanding in powers of the impurity concentration.

In his theory there is only one parameter which describes the impurity distribution, namely, the mean distance between impurities, related inversely to their concentration. Impurity-impurity coupling is taken into account, but only as a small perturbation on an infinitely dilute system.

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[1] G. F. Koster and J. C. Stater, Phys. Rev. 95, 1167 (1954).
[2] G. F. Koster, Phys. Rev. 95, 1436 (1954).
[3] G. F. Koster and J. C. Stater, Phys. Rev. 96, 1208 (1954).
[4] R. E. Merrifield, J. Chem. Phys. 38, 920 (1963).
[5] S. Takeno, J. Chem. Phys. 44, 853 (1966).
[6] I. M. Lifshitz, Advan. Phys. 3, 483 (1964).